Training deep resistive networks with equilibrium propagation

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Computing with Physical Systems

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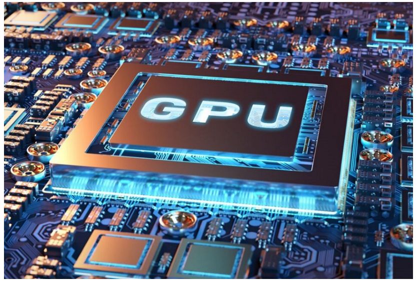
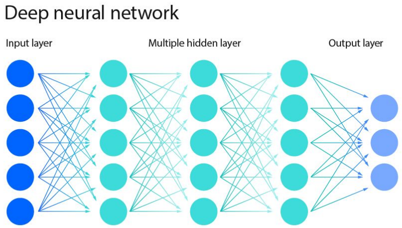
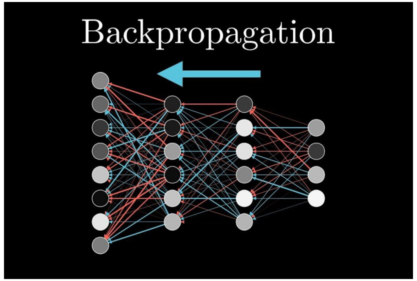
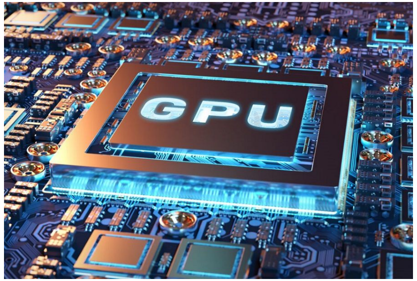
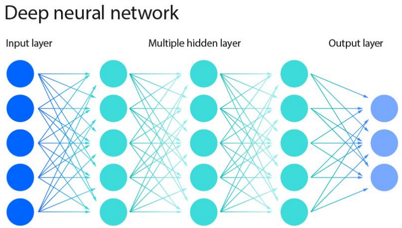
Deep Learning Computing Paradigm

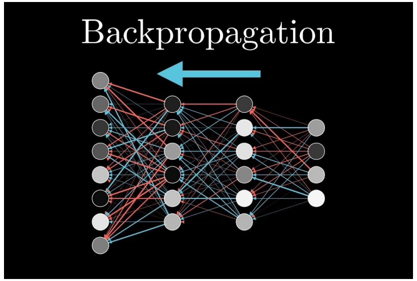


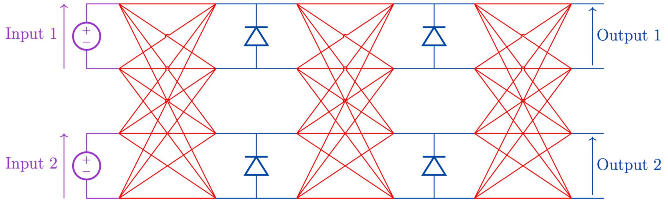




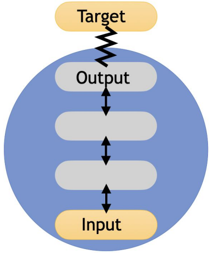








Deep resistive network



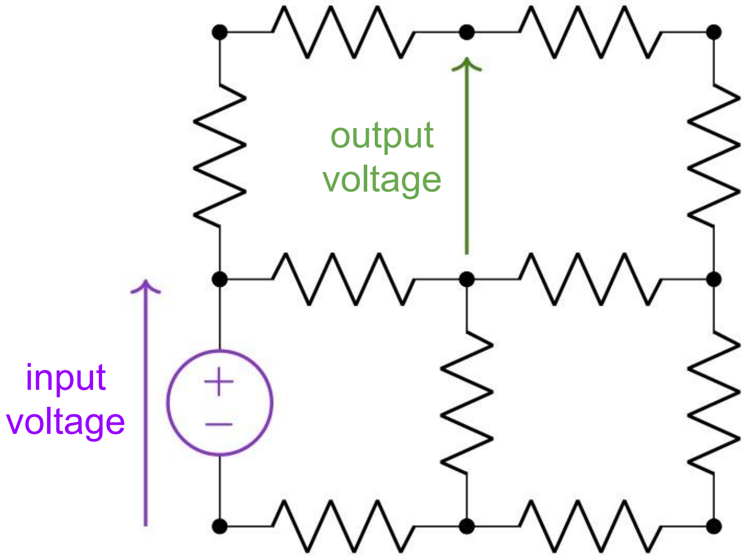
Equilibrium propagation

What is a deep resistive network?

Computing with a Resistor Network

Circuit elements:

- voltage sources (inputs)



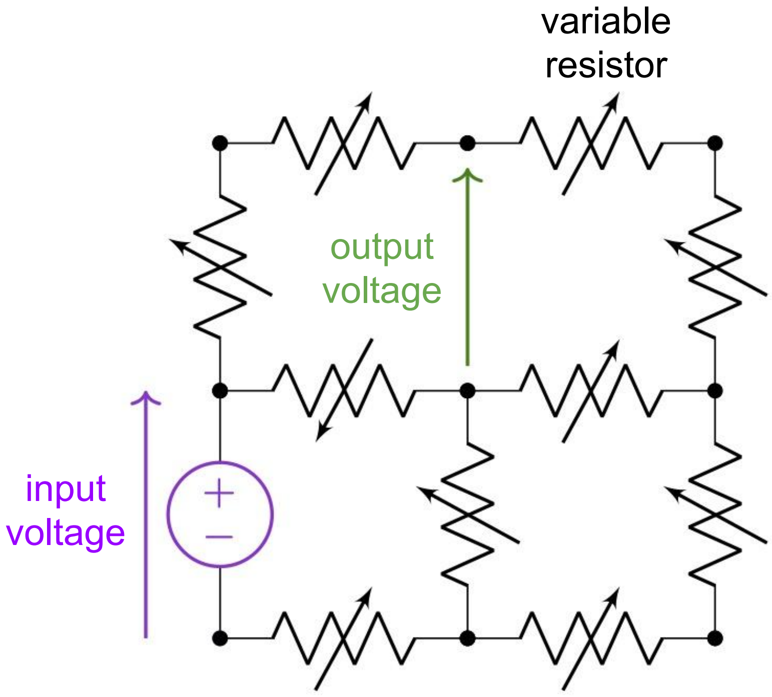
$$\mathrm{output} = F(\mathrm{input})$$

Reference: Scellier and Mishra. “A universal approximation theorem for nonlinear resistive networks” (2023)

Computing with a Resistor Network

Circuit elements:

- voltage sources (inputs)
- variable resistors (trainable weights)



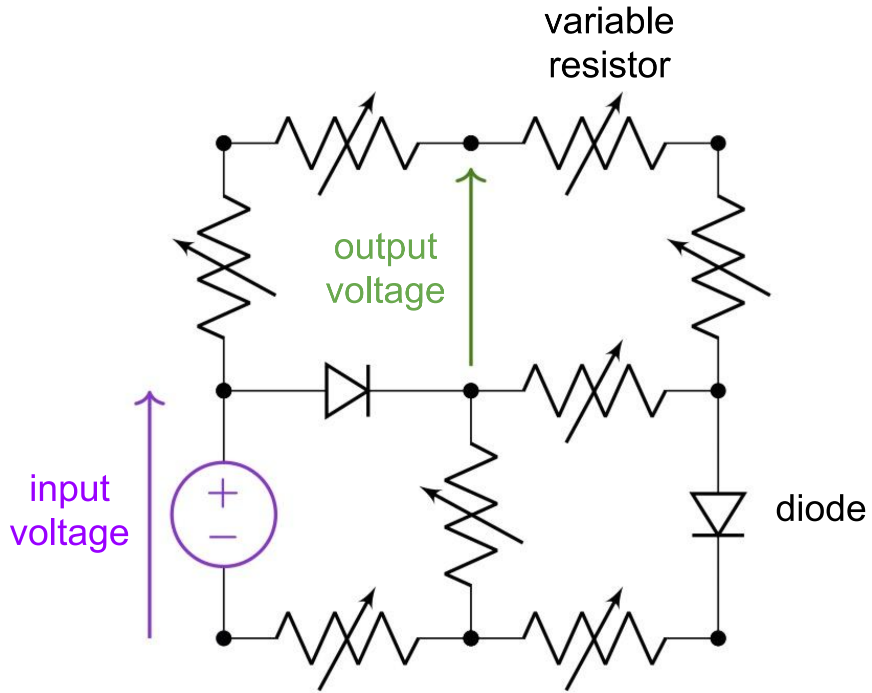
$$\text{output} = F_{\text{conductances}}(\text{input})$$

Reference: Scellier and Mishra. “A universal approximation theorem for nonlinear resistive networks” (2023)

Computing with a Resistor Network

Circuit elements:

- voltage sources (inputs)
- variable resistors (trainable weights)
- diodes (nonlinearities)



$$\text{output} = F_{\text{conductances}}(\text{input})$$

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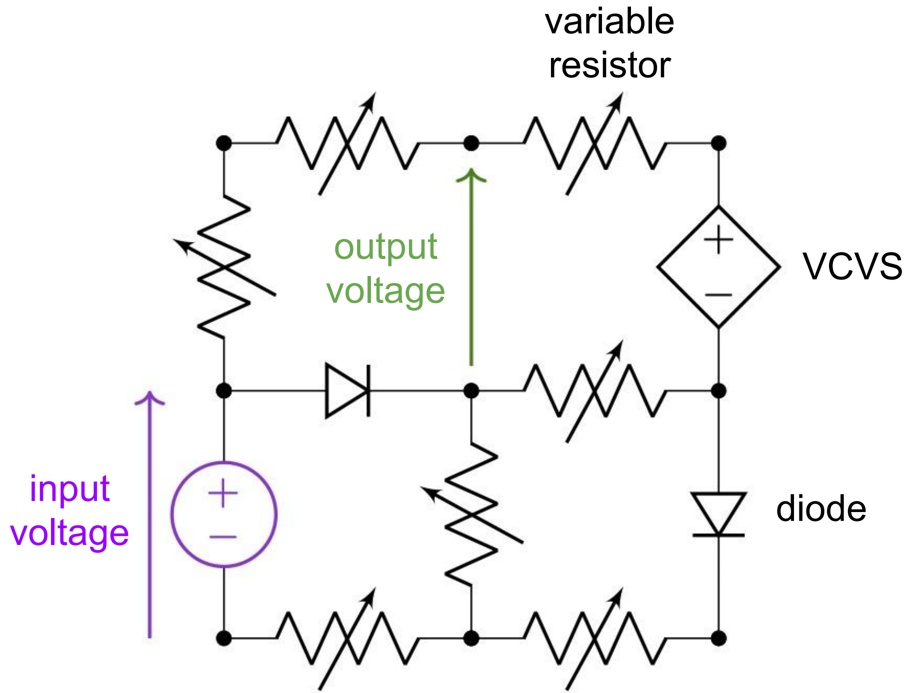
Computing with a Resistor Network

Circuit elements:

- voltage sources (inputs)
- variable resistors (trainable weights)
- diodes (nonlinearities)
- VCVS (amplification)

Such electrical circuits are
universal function approximators



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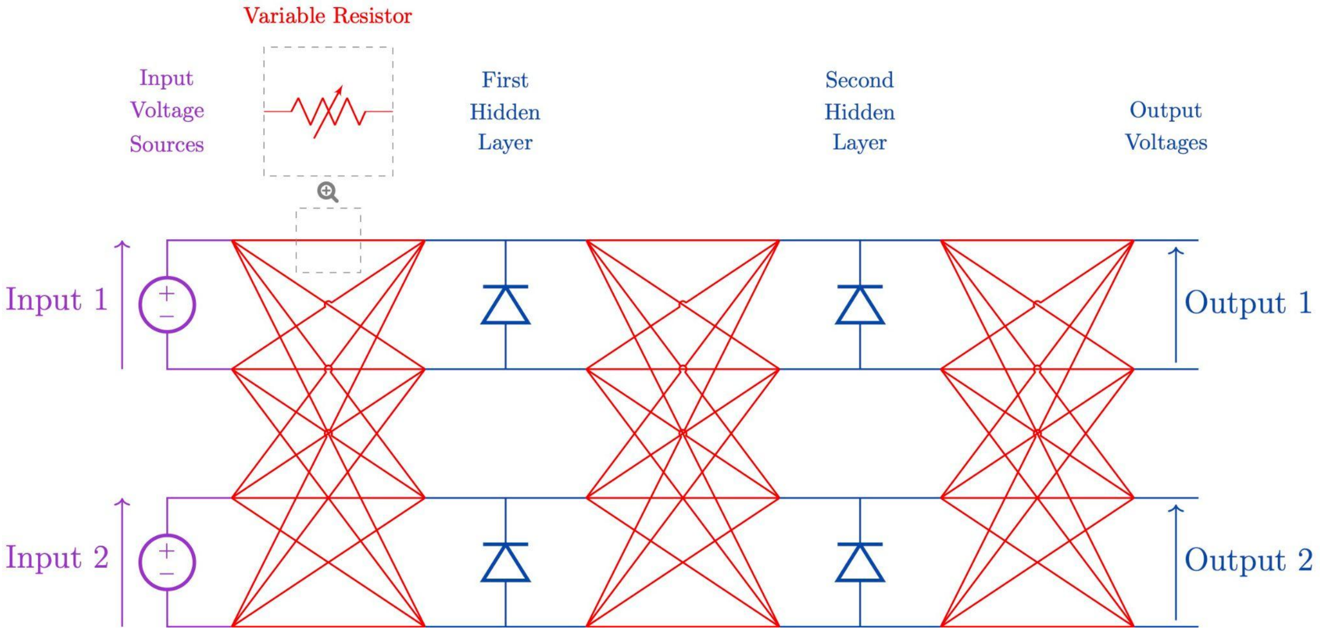
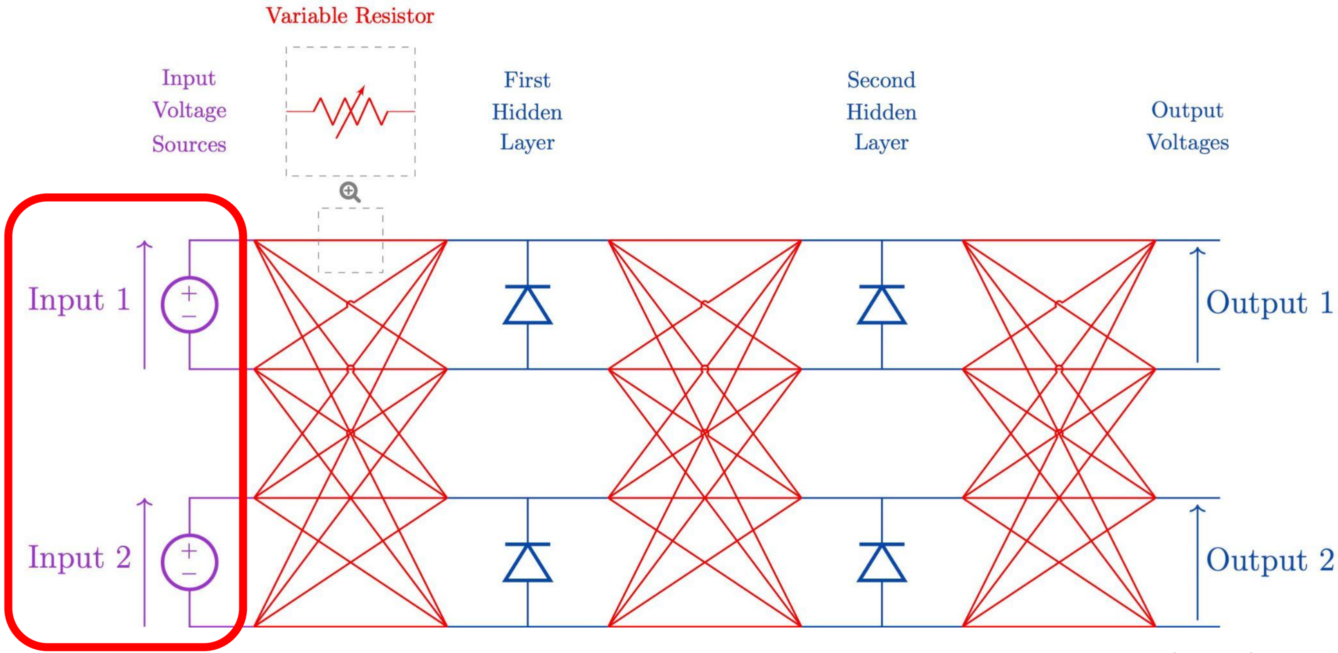
Deep Resistive Network



Reference: Kendall et al. “Training end-to-end analog neural networks with equilibrium propagation” (2020)

Deep Resistive Network

- input voltage sources



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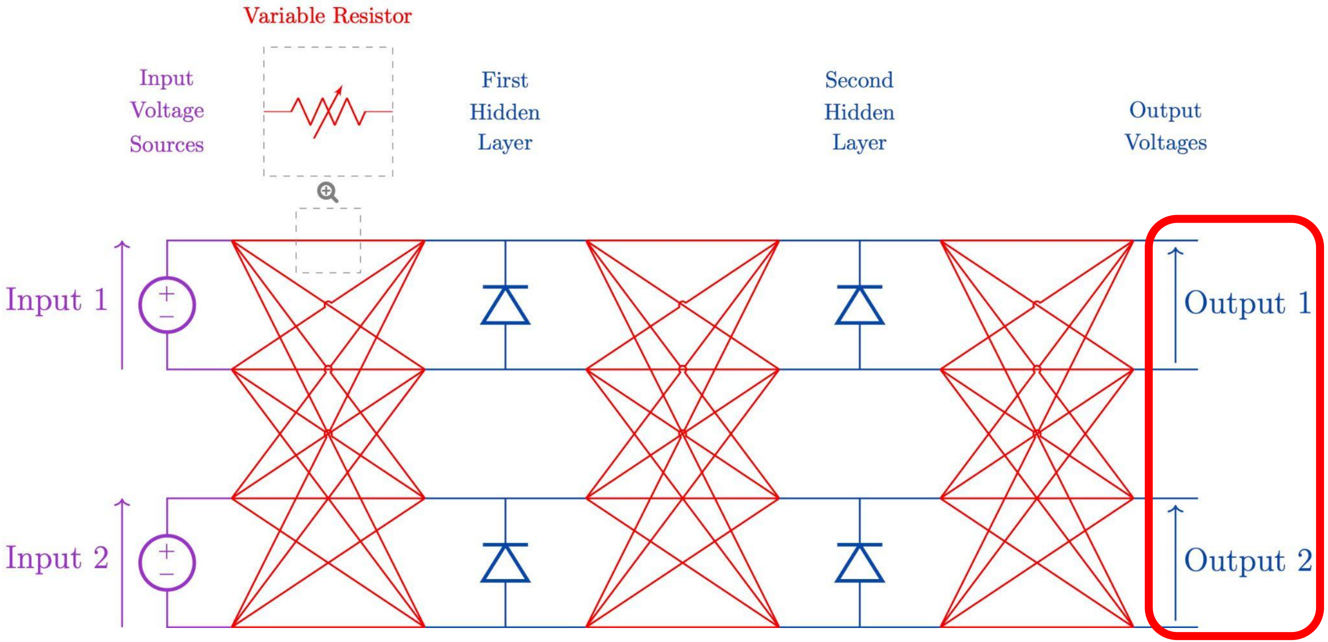
Deep Resistive Network

- input voltage sources
- output voltages



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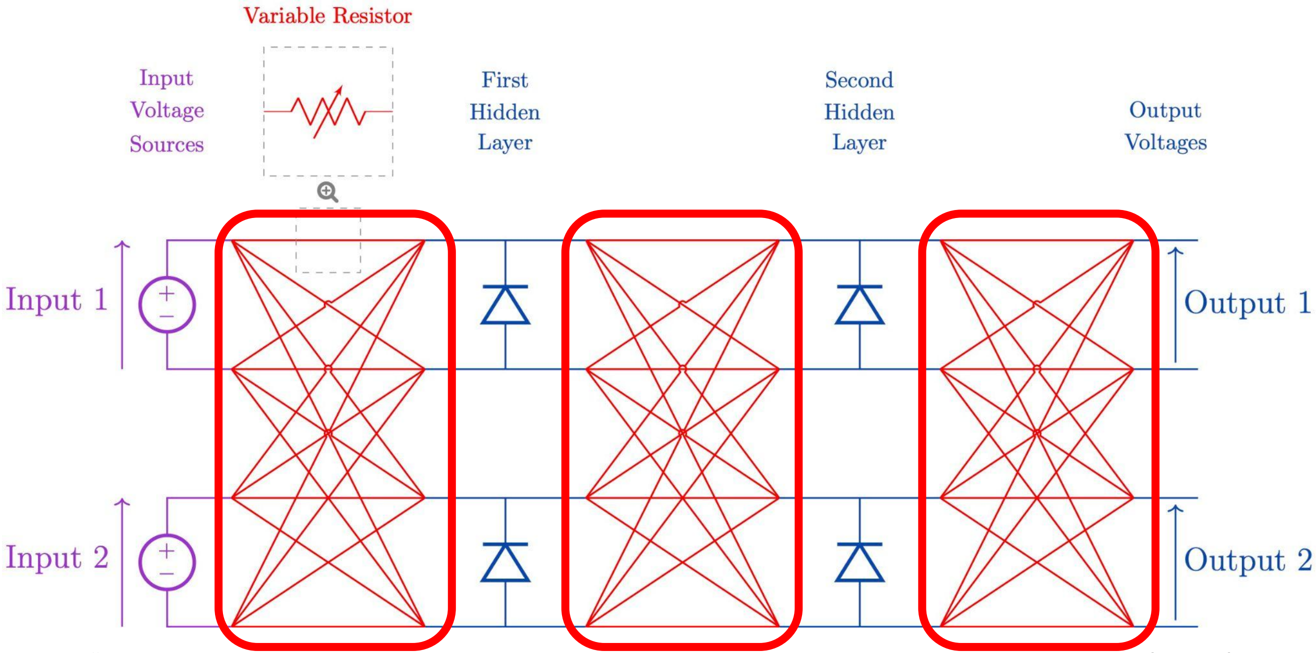
Deep Resistive Network

- input voltage sources
- output voltages
- crossbar arrays of variable resistors (linear trainable weights)



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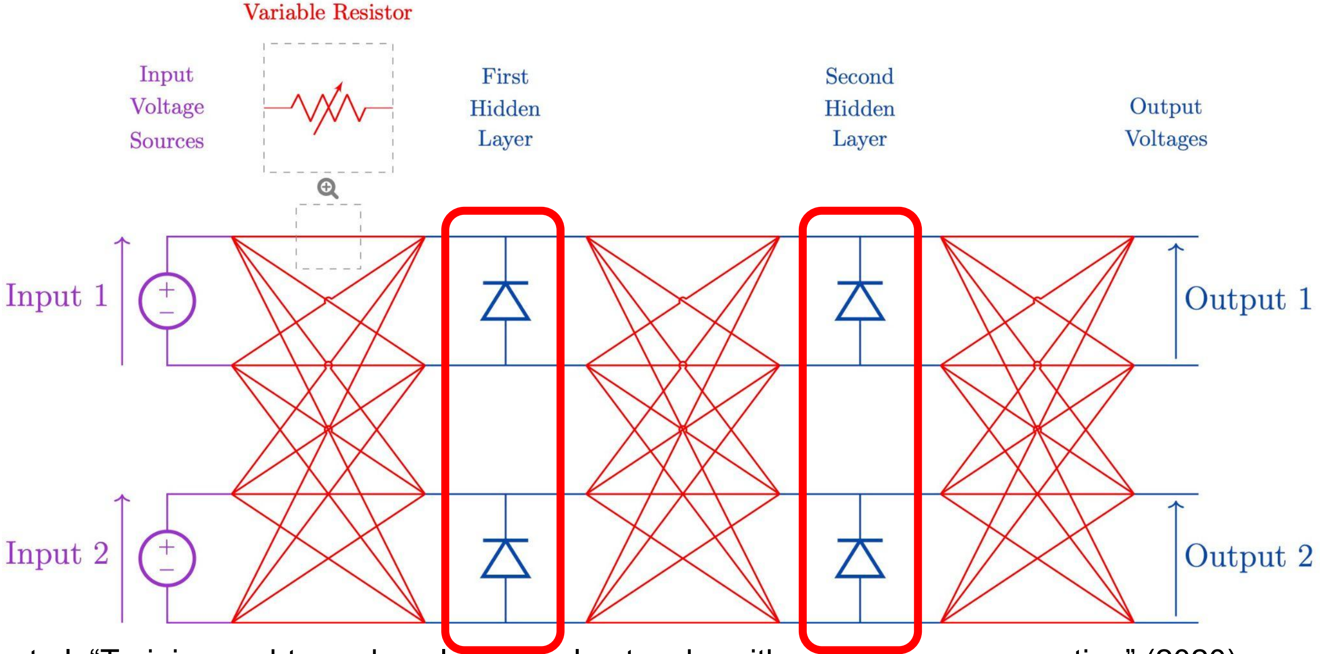
Deep Resistive Network

- input voltage sources
- output voltages
- crossbar arrays of variable resistors (linear trainable weights)
- diodes (nonlinearities)



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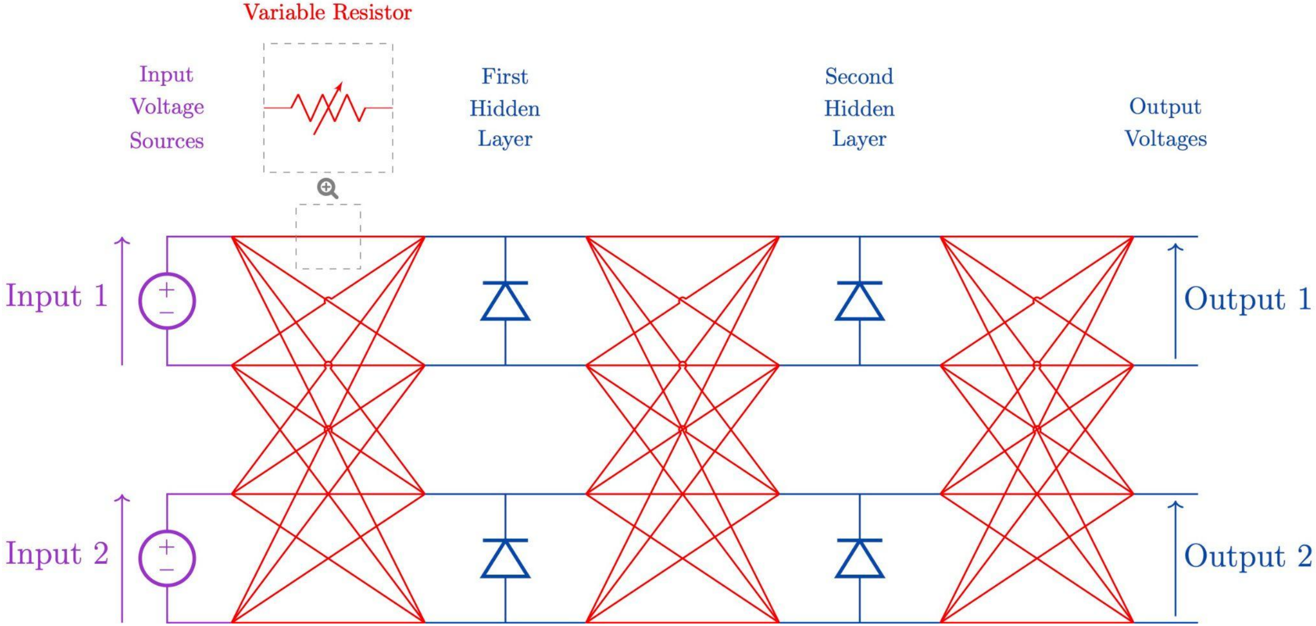
Deep Resistive Network

Properties:

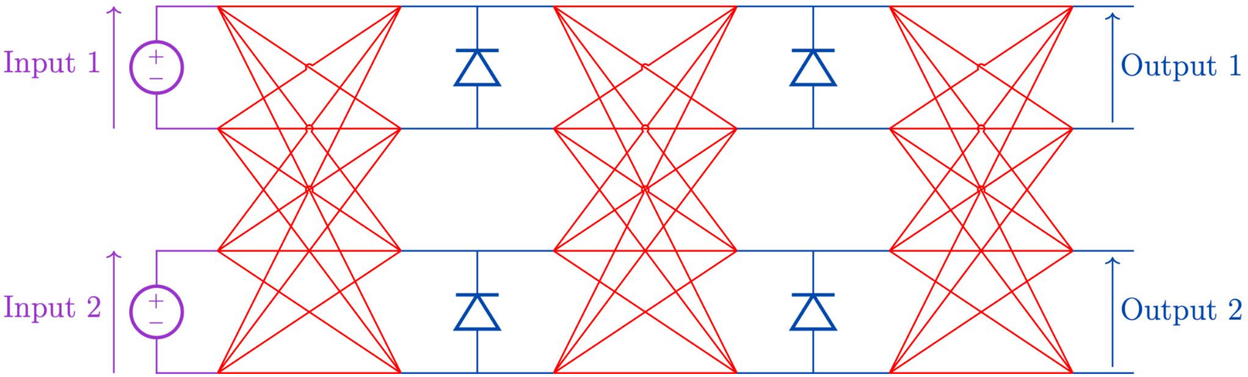
- universal function approximator
- can be trained via equilibrium propagation



Reference: Kendall et al. “Training end-to-end analog neural networks with equilibrium propagation” (2020)

What is equilibrium propagation?

Equilibrium Propagation in a Deep Resistive Network

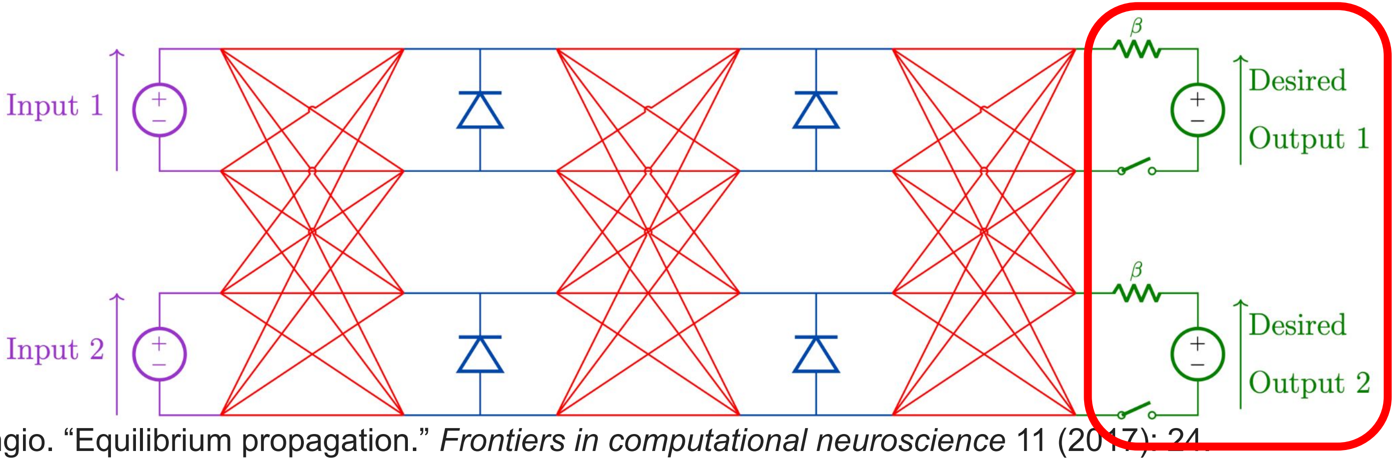


References:
Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.
Kendall et al. “Training end-to-end analog neural networks with equilibrium propagation” (2020)

Equilibrium Propagation in a Deep Resistive Network

EP learning requires augmenting the network:

- for each pair of output nodes, add switch + voltage source + resistor



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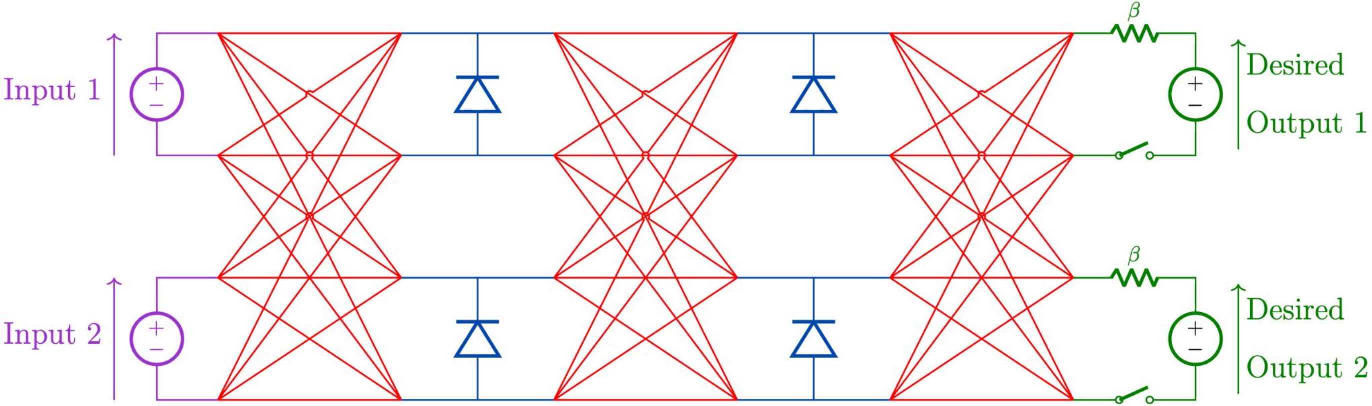
Equilibrium Propagation in a Deep Resistive Network

Training procedure:



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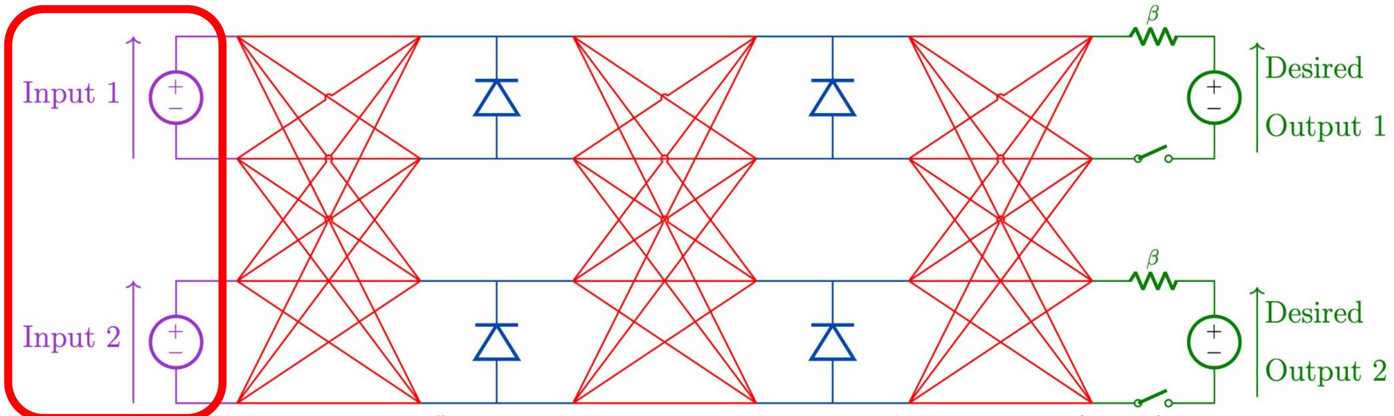
Equilibrium Propagation in a Deep Resistive Network

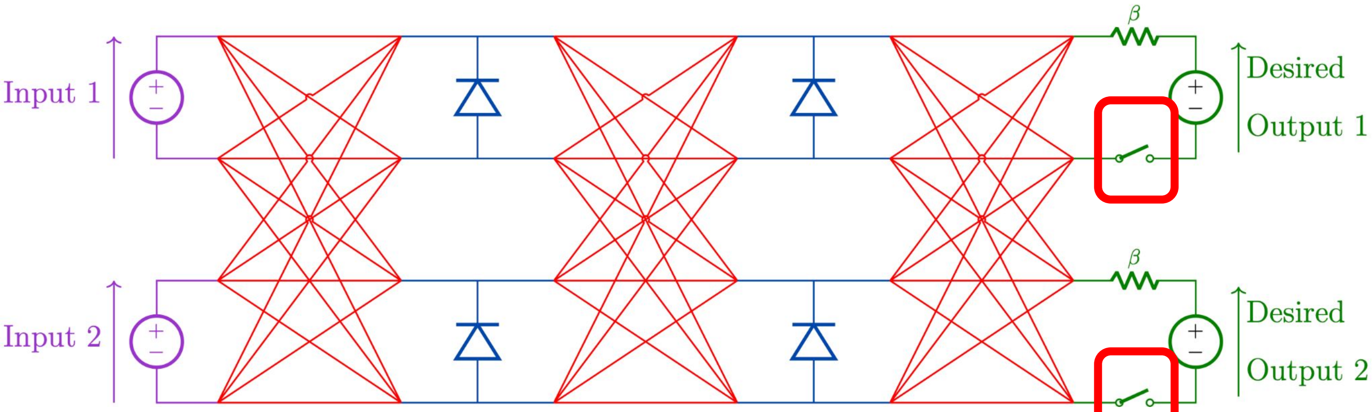
Training procedure:
1. Set input voltages.



References:
Scellier and Bengio. "Equilibrium propagation." *Frontiers in computational neuroscience* 11 (2017): 24.
Kendall et al. "Training end-to-end analog neural networks with equilibrium propagation" (2020)

Equilibrium Propagation in a Deep Resistive Network

<u>Training procedure:</u>
1. Open output switches.



<u>References</u>:
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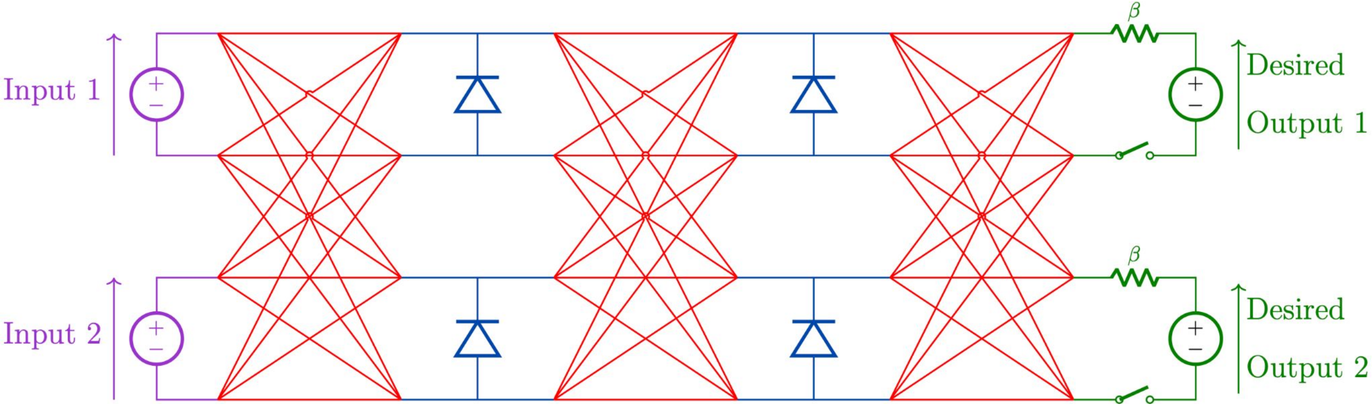
Kendall et al. "Training end-to-end analog neural networks with equilibrium propagation" (2020)

Equilibrium Propagation in a Deep Resistive Network

<u>Training procedure:</u>
1. Open output switches. Observe the steady state (“free state”).



<u>References</u>:
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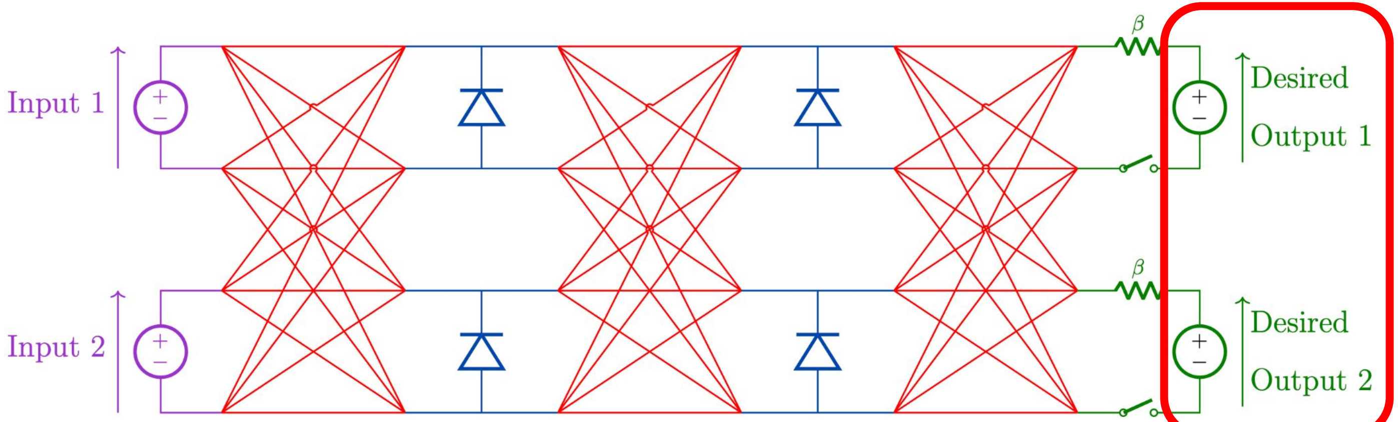
Kendall et al. “Training end-to-end analog neural networks with equilibrium propagation” (2020)

Equilibrium Propagation in a Deep Resistive Network

<u>Training procedure:</u>
1. Open output switches: “free state”
2. Set desired output voltages.



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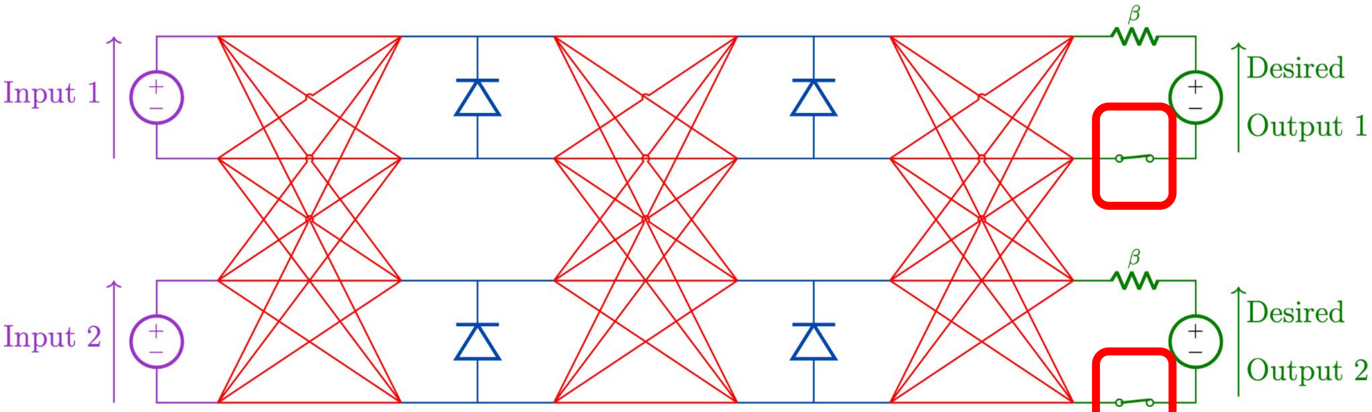
Equilibrium Propagation in a Deep Resistive Network

<u>Training procedure:</u>
1. Open output switches: “free state”
2. Close output switches.



<u>References</u>:
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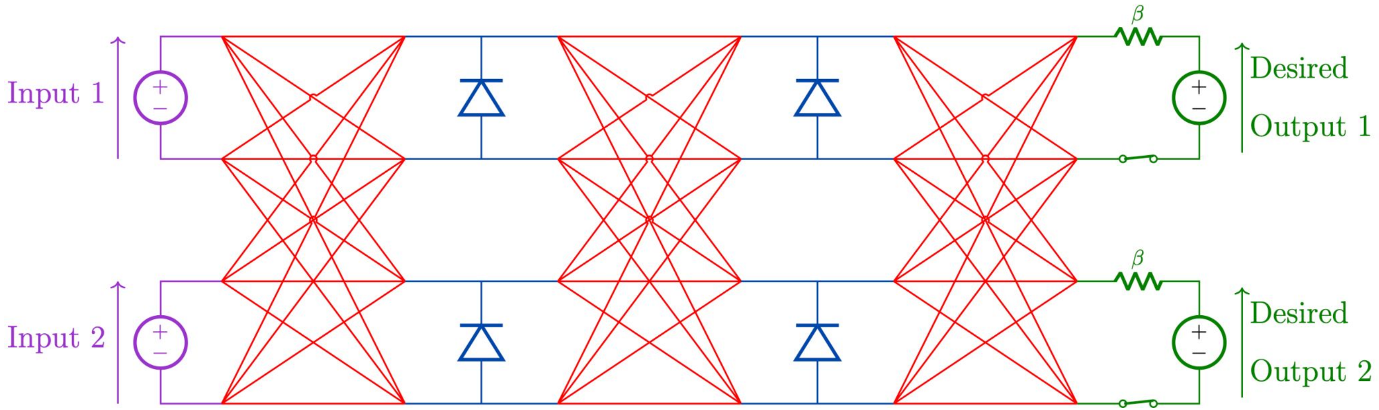
Equilibrium Propagation in a Deep Resistive Network

<u>Training procedure:</u>
1. Open output switches: “free state”
2. Close output switches. Observe the new steady state (“nudged state”).



<u>References</u>:
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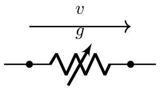
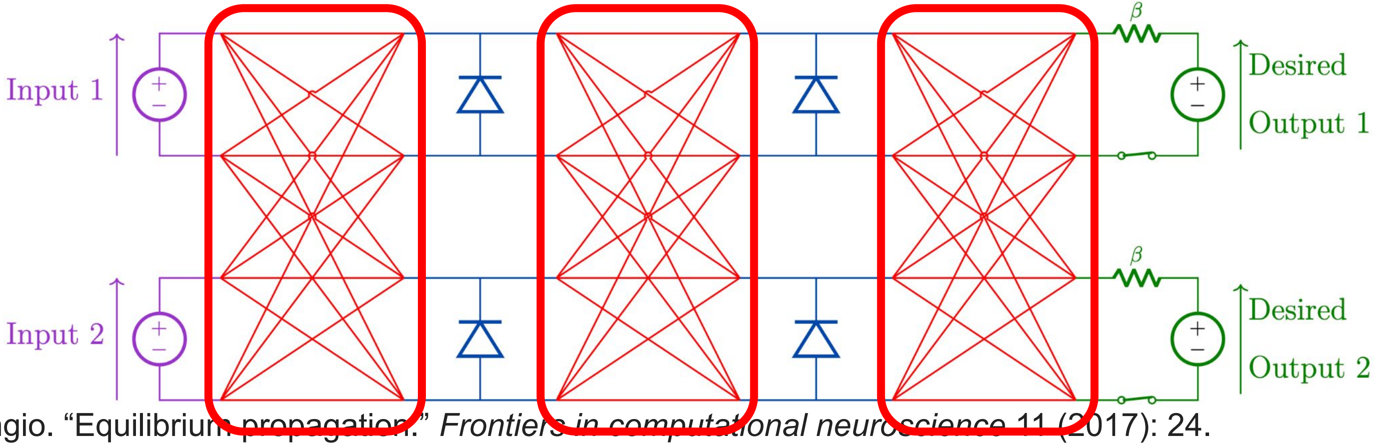
Equilibrium Propagation in a Deep Resistive Network

<u>Training procedure:</u>
1. Open output switches: “free state”
2. Close output switches: “nudged state”
3. For each resistor 

update the conductance g $\Delta g = \frac{\eta}{2\beta}\left(v_{\text{free}}^2 - v_{\text{nudged}}^2\right)$



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Equilibrium Propagation in a Deep Resistive Network

Training procedure:
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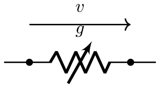
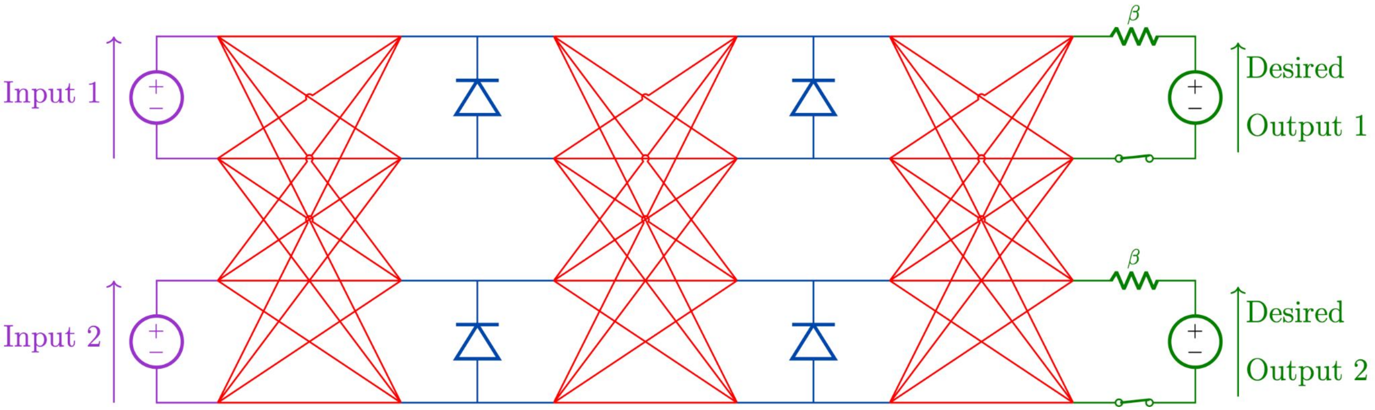
3. For each resistor  update the conductance g

$$\boxed{\Delta g = \frac{\eta}{2\beta}\left(v_{\text{free}}^2 - v_{\text{nudged}}^2\right)}$$

local in space



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Equilibrium Propagation in a Deep Resistive Network

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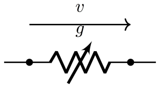
3. For each resistor 

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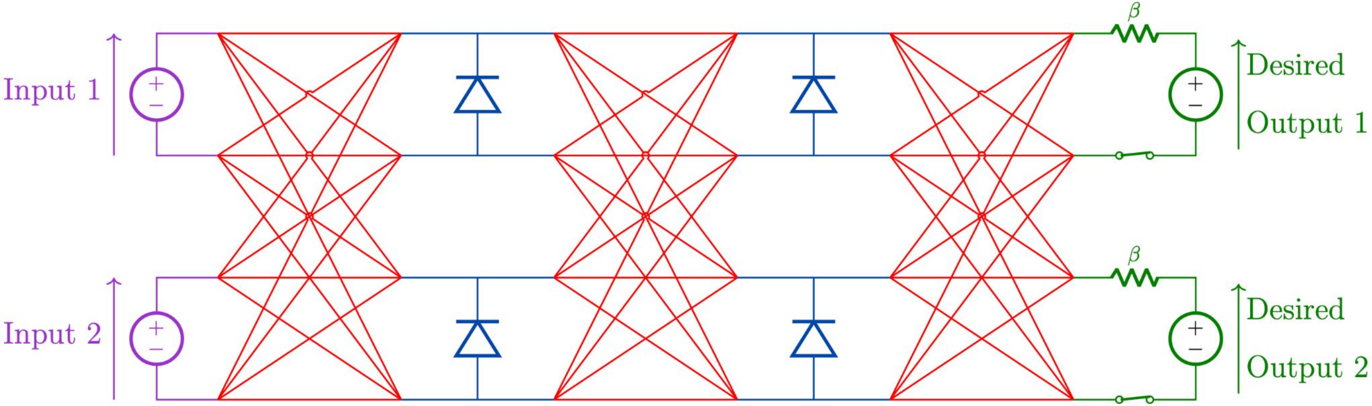
<u>Theorem</u>: weight updates approximate gradient descent on the MSE

$$\Delta g = -\eta \nabla_g \text{MSE} + O(\beta)$$



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Equilibrium Propagation in a Deep Resistive Network

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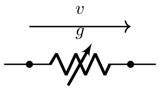
3. For each resistor 

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<u>Bottom line</u>: **local learning rule** performs **stochastic gradient descent (SGD)**

Equilibrium Propagation in a Deep Resistive Network

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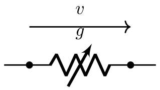
3. For each resistor 

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Theorem: weight updates approximate gradient descent on the cost function

$$\Delta g = -\eta \nabla_g C + O(\beta)$$

Bottom line: **local learning rule** performs **stochastic gradient descent (SGD)**

Remarks:
1. applicable to any cost function C (not just the MSE)

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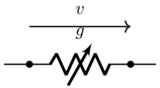
3. For each resistor 

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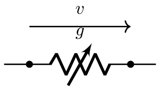
3. For each resistor 

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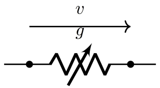
3. For each resistor 

update the conductance g $\Delta g = \frac{\eta}{2\beta}\left(v_{\text{free}}^2 - v_{\text{nudged}}^2\right)$

Theorem: weight updates perform gradient descent on a surrogate function

$$\Delta g = -\eta \nabla_g \mathcal{L}_\beta$$ where $$\mathcal{L}_\beta = C + O(\beta)$$

Bottom line: **local learning rule** performs **stochastic gradient descent (SGD)**

Remarks:
1. applicable to any cost function C (not just the MSE)
2. applicable to any network topology (not just a DRN)

Equilibrium Propagation in a Deep Resistive Network

<u>Sketch of the proof:</u>

- Equilibrium propagation generally applies to systems whose steady state minimizes a functional (variational principle) [1,2]

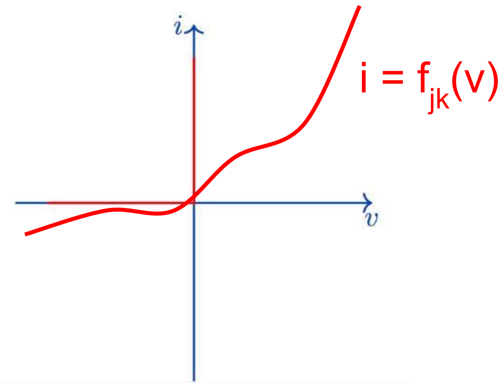
- Nonlinear resistive networks minimize a functional called *co-content* [3]

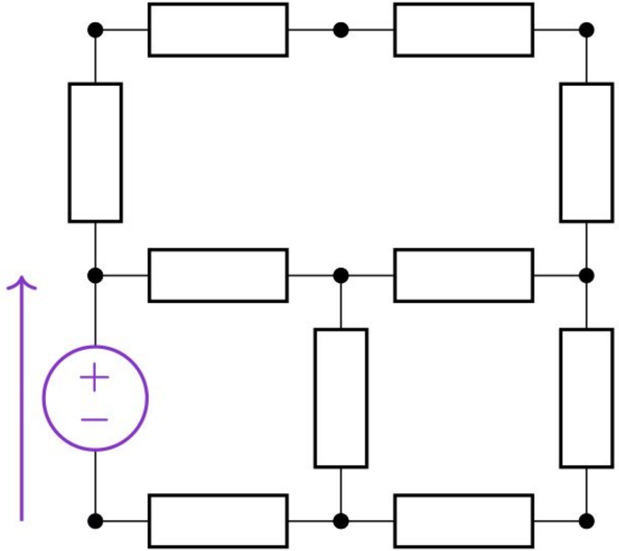
<u>References</u>

[1] Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.
[2] Scellier. “A deep learning theory for neural networks grounded in physics.” *PhD thesis, Université de Montréal* (2021).
[3] Millar. "CXVI. Some general theorems for non-linear systems possessing resistance." The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 42.333 (1951): 1150-1160.

What is the co-content?

i-v curve of branch (j,k)





Co-content of branch (j,k)

$$E_{jk} = \int_0^{v_{jk}} f_{jk}(u)du$$

Total co-content of the circuit

$$E_{\text{total}} = \sum_{j,k} E_{jk}$$

References

Millar. "CXVI. Some general theorems for non-linear systems possessing resistance." The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 42.333 (1951): 1150-1160.

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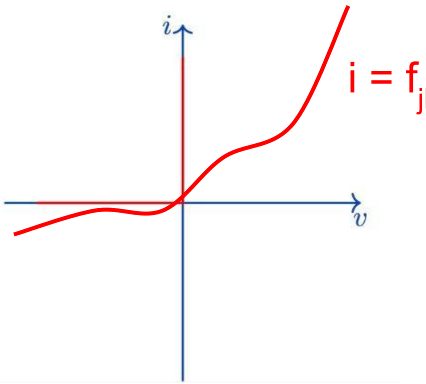
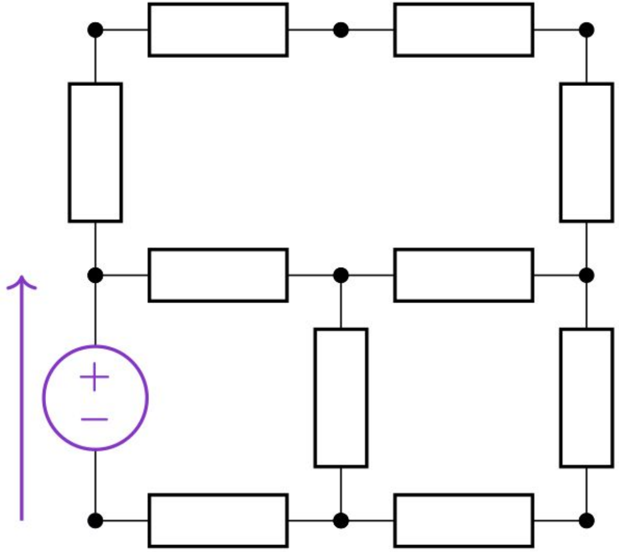




Co-content of branch (j,k)

$$E_{jk} = \int_0^{v_{jk}} f_{jk}(u)du$$

Total co-content of the circuit

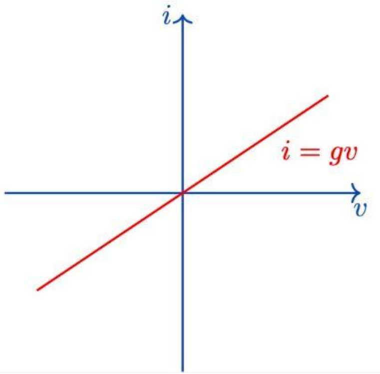
$$E_{\text{total}} = \sum_{j,k} E_{jk}$$

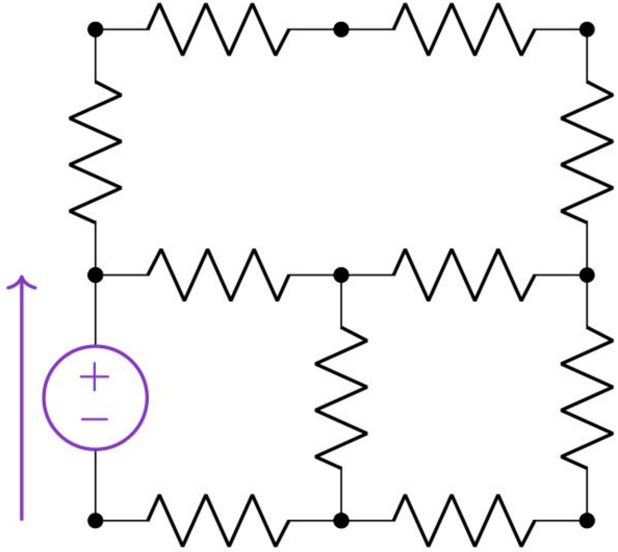
branch equation + KVL

KCL

$$\delta E_{\text{total}} = \sum_{j,k} \delta E_{jk} = \sum_{j,k} f_{jk}\, \delta v_{jk} \overset{\downarrow}{=} \sum_{j,k} i_{jk}(\delta v_j - \delta v_k) = \sum_j \delta v_j \left(\sum_k i_{jk} \right) \overset{\downarrow}{=} 0$$

Linear resistor





Co-content of linear resistor

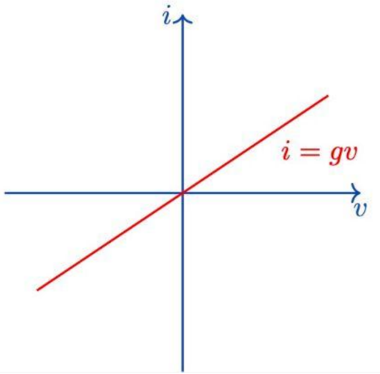
$$E = \frac{1}{2} g v^2$$

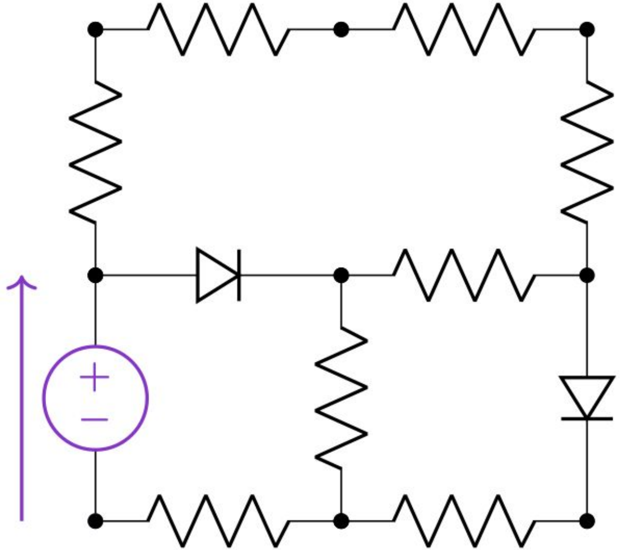
(half the power dissipation)

Total co-content of the circuit

$$E_{\text{total}} = \frac{1}{2} \sum_{j,k} g_{jk} v_{jk}^2$$

Linear resistor





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Total co-content of the circuit

$$E_{\text{total}} = \frac{1}{2} \sum_{j,k} g_{jk} v_{jk}^2 + E_{\text{diodes}}$$

Equilibrium Propagation Learning Rule

Learning rule

$$\Delta g = \frac{\eta}{2\beta}\left(\underbrace{v_{\text{free}}^2}_{\partial_g E(\text{free})} - \underbrace{v_{\text{nudged}}^2}_{\partial_g E(\text{nudged})}\right)$$

Co-content

$$E = \frac{1}{2}\sum_{\text{resistor}} g v^2 + E_{\text{diodes}}$$

Reference: Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.

Equilibrium Propagation Learning Rule

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$$\Delta g = \frac{\eta}{\beta}\left(\partial_g E(\text{free}) - \partial_g E(\text{nudged})\right)$$

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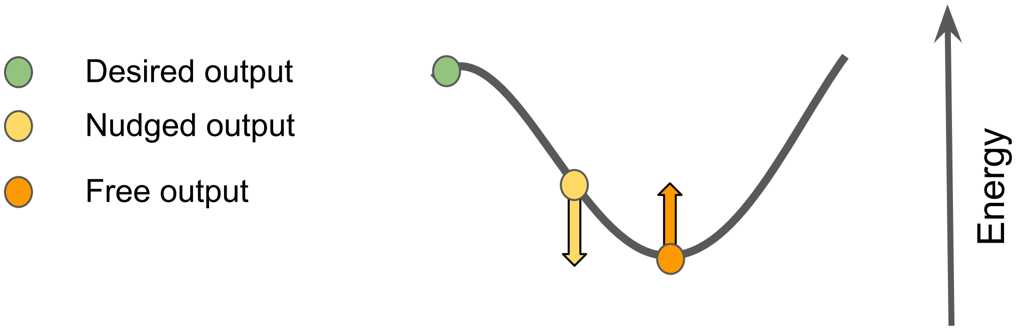
Equilibrium Propagation Learning Rule

Learning rule

$$\Delta w = \frac{\eta}{\beta} \left(\partial_w E(\text{free}) - \partial_w E(\text{nudged}) \right)$$

Reference: Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.

Equilibrium Propagation Learning Rule



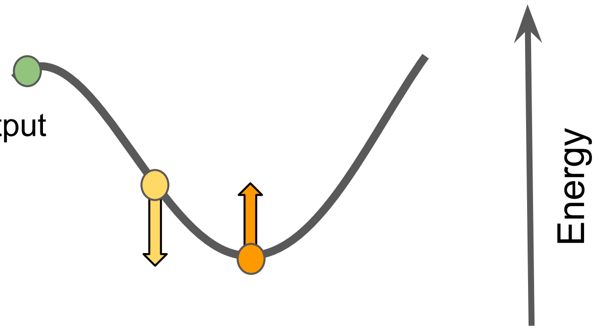
Learning rule

$$\Delta w = \frac{\eta}{\beta}\left(\partial_w E(\mathrm{free}) - \partial_w E(\mathrm{nudged})\right)$$

Reference: Scellier and Bengio. "Equilibrium propagation." *Frontiers in computational neuroscience* 11 (2017): 24.

Variants of Equilibrium Propagation (EP)

- Desired output
- Positively-perturbed output
- Free output



Positively-perturbed EP
pull outputs **towards** desired outputs

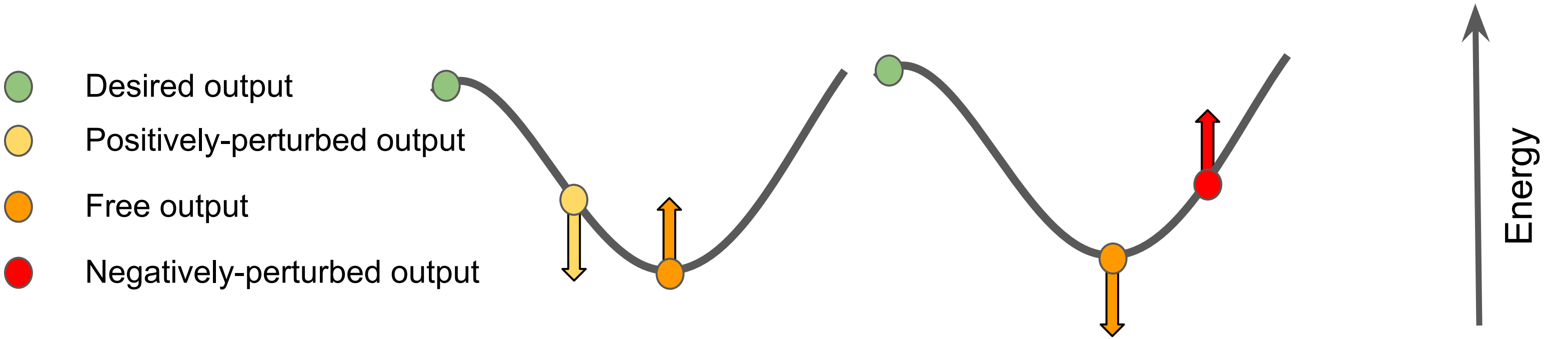
References:

Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.

Laborieux et al. “Scaling equilibrium propagation to deep ConvNets by reducing its gradient estimator bias” (2021)

Scellier et al. “Energy-based learning algorithms: a comparative study” NeurIPS (2023)

Variants of Equilibrium Propagation (EP)



Positively-perturbed EP	Negatively-perturbed EP
pull outputs **towards** desired outputs	**push** outputs **away from** desired outputs

References:

Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.

Laborieux et al. “Scaling equilibrium propagation to deep ConvNets by reducing its gradient estimator bias” (2021)

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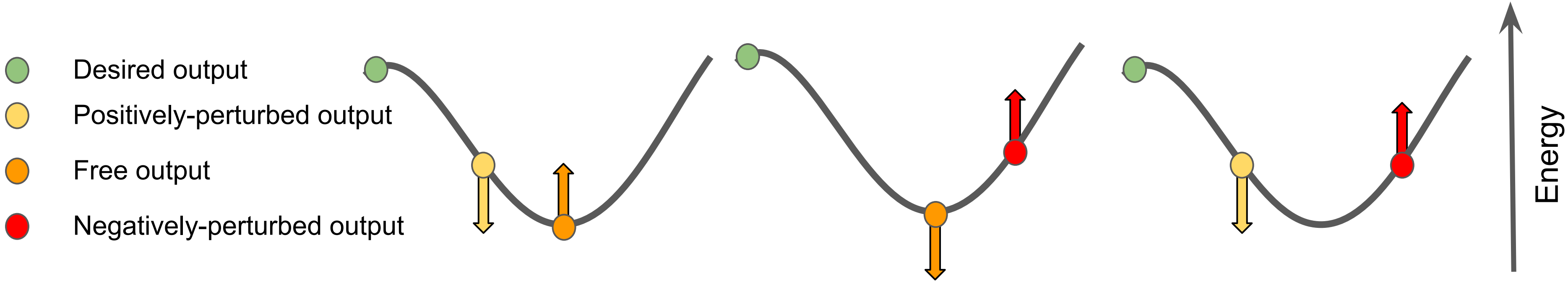
Variants of Equilibrium Propagation (EP)



Positively-perturbed EP	Negatively-perturbed EP	Centered EP
pull outputs **towards** desired outputs	**push** outputs **away from** desired outputs	➔ one **positive** perturbation ➔ one **negative** perturbation

References:
Scellier and Bengio. "Equilibrium propagation." *Frontiers in computational neuroscience* 11 (2017): 24.
Laborieux et al. "Scaling equilibrium propagation to deep ConvNets by reducing its gradient estimator bias" (2021)
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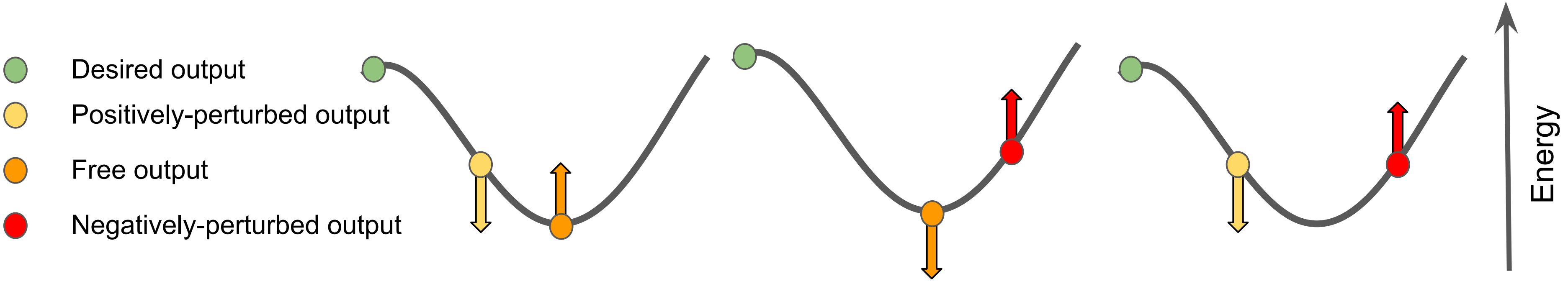
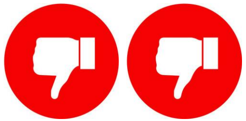
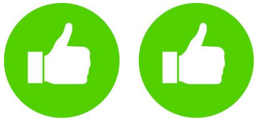
Variants of Equilibrium Propagation (EP)



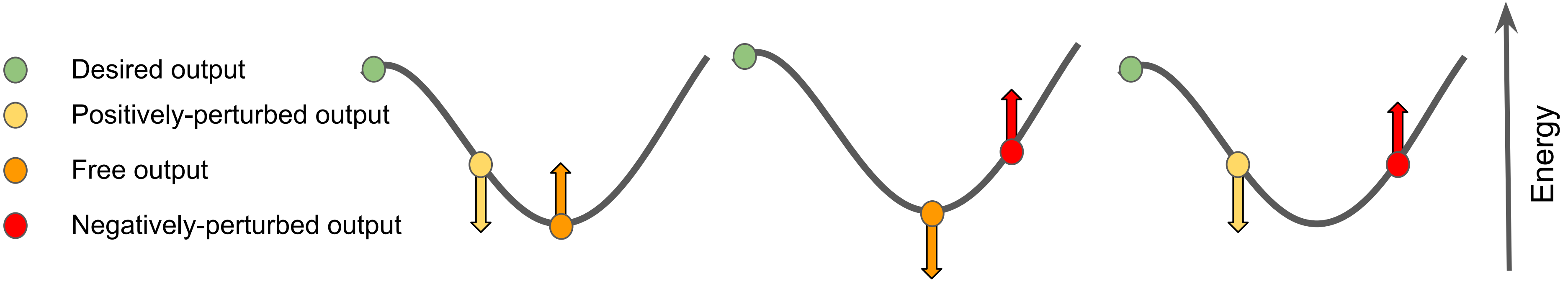
Positively-perturbed EP	Negatively-perturbed EP	Centered EP
pull outputs **towards** desired outputs	**push** outputs **away from** desired outputs	➔ one **positive** perturbation ➔ one **negative** perturbation

Performance in practice?





Variants of Equilibrium Propagation (EP)



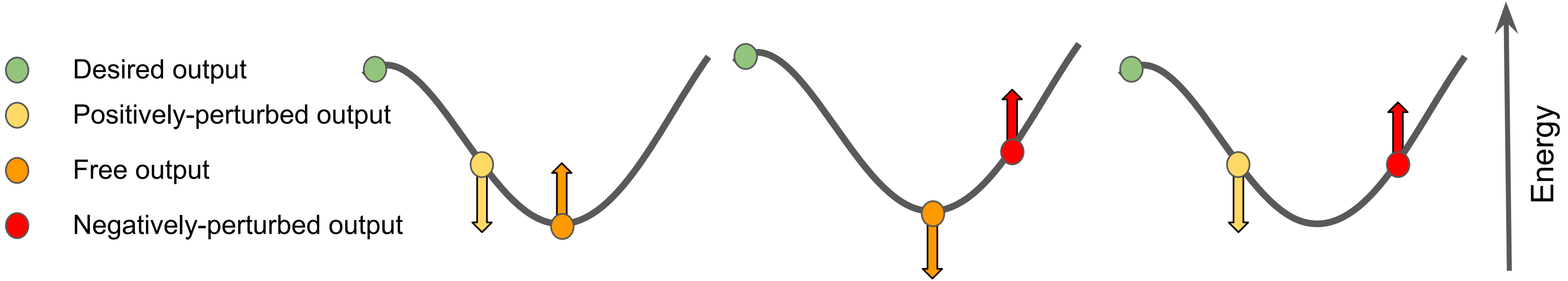
	Positively-perturbed EP	Negatively-perturbed EP	Centered EP

<u>Theorem</u>: EP computes the gradient of a surrogate function that approximates the cost function

$$\Delta w = -\eta \nabla_w \mathcal{L}_\beta \qquad\qquad \mathcal{L}_\beta = C + O(\beta)$$

<u>Reference:</u> Scellier et al. “Energy-based learning algorithms: a comparative study” NeurIPS (2023)

Variants of Equilibrium Propagation (EP)

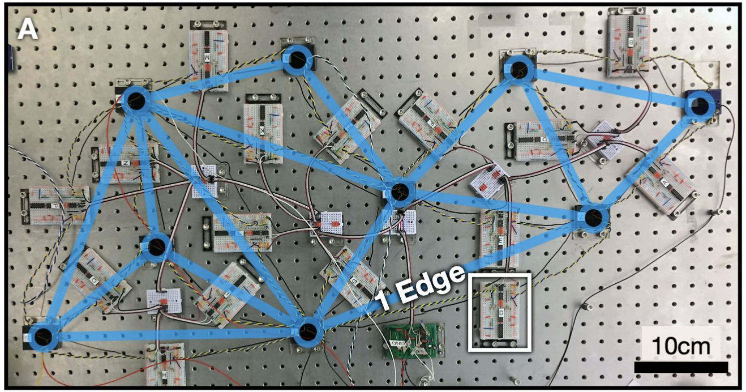


	Positively-perturbed EP	Negatively-perturbed EP	Centered EP
Properties of the surrogate function	$L_{P\text{-}EP} = C +$ **O(β)** $L_{P\text{-}EP} \leq C$ (**lower bound**)	$L_{N\text{-}EP} = C +$ **O(β)** $L_{N\text{-}EP} \geq C$ (**upper bound**)	$L_{C\text{-}EP} = C +$ **O(β²)**

<u>Reference:</u> Scellier et al. "Energy-based learning algorithms: a comparative study" NeurIPS (2023)

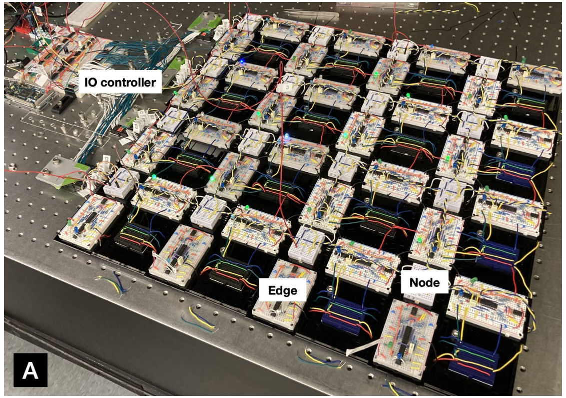
Hardware Experiments

Linear resistor network [1]



- Network of 9 nodes and 16 edges
- Task: classify the Iris dataset

Nonlinear resistive network [2]

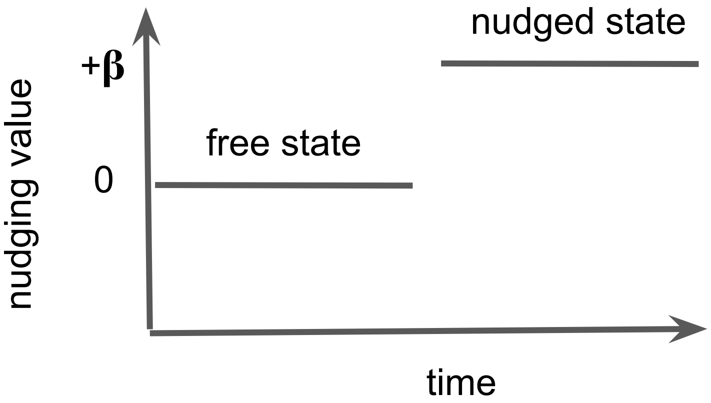


- Network of 32 (transistor-based) twin edges
- Tasks: XOR and nonlinear regression

Caveat: use two copies of the same network
(one for the free state, one for the nudge state)

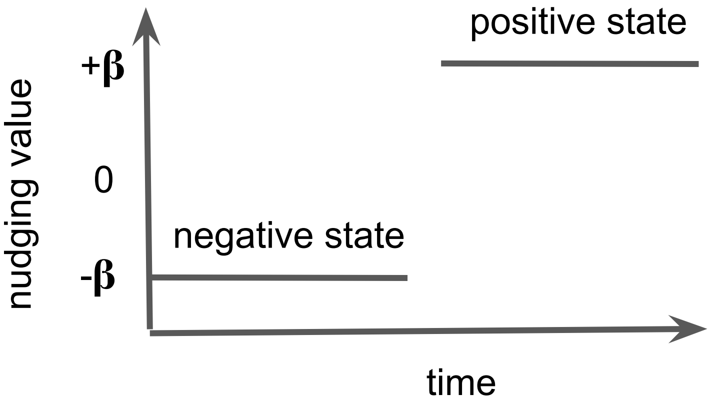
References
[1] Dillavou et al. "Demonstration of decentralized physics-driven learning." Physical Review Applied (2022)
[2] Dillavou et al. "Circuits that train themselves: decentralized, physics-driven learning." (2023)

Equilibrium propagation

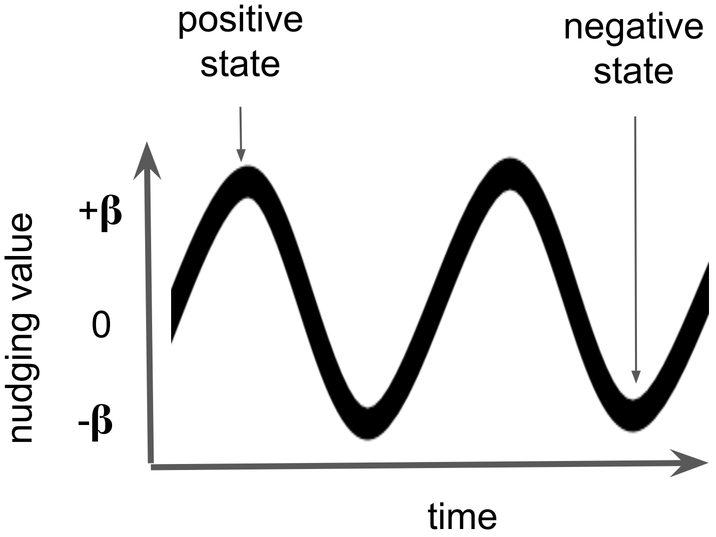


Learning rule $\Delta g = \frac{\eta}{2\beta}\left(v_{\text{free}}^2 - v_{\text{nudged}}^2\right)$

Equilibrium propagation



Frequency propagation [1]



Learning rule $$\Delta g = \frac{\eta}{4\beta}\left(v_{\mathrm{neg}}^2 - v_{\mathrm{pos}}^2\right) = \frac{\eta}{4\beta}\left(v_{\mathrm{neg}} + v_{\mathrm{pos}}\right)\left(v_{\mathrm{neg}} - v_{\mathrm{pos}}\right)$$

state(t) = mean + amplitude * sin(ωt) + O($\boldsymbol{\beta}^2$)

Reference
[1] Anisetti et al. "Frequency propagation: Multi-mechanism learning in nonlinear physical networks." (2022)

Simulations of deep resistive networks

Task: train a DRN to classify MNIST digits

	Network size (number of parameters)	Number of epochs	Total duration	Test error rate
SPICE simulations [1]	0.16M	10	1 week	3.43%

References
[1] Kendall et al. “Training end-to-end analog neural networks with equilibrium propagation” (2020)

Simulations of deep resistive networks

Task: train a DRN to classify MNIST digits

	Network size (number of parameters)	Number of epochs	Total duration	Test error rate
SPICE simulations [1]	0.16M	10	1 week	3.43%
DRN simulator [2,3]	51M	50	6 hours	1.40%
	320x larger	5x more	28x shorter	

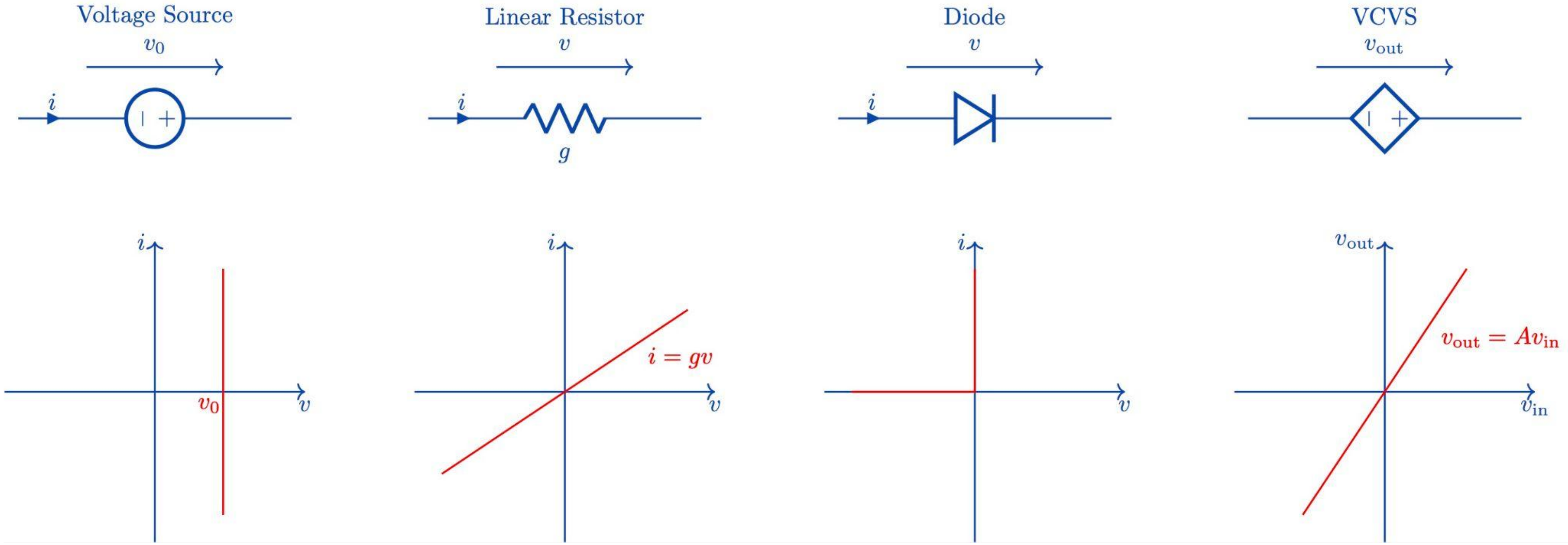
The "network size" to "epoch duration" ratio is 45000 times larger

References
[1] Kendall et al. "Training end-to-end analog neural networks with equilibrium propagation" (2020)
[2] "A fast algorithm to simulate nonlinear resistive networks". To appear.
[3] Code will be available at https://github.com/rain-neuromorphics/energy-based-learning

Simulations of deep resistive networks

Main idea:

- assumption: the circuit elements are **ideal**



Reference: “A fast algorithm to simulate nonlinear resistive networks”. To appear.

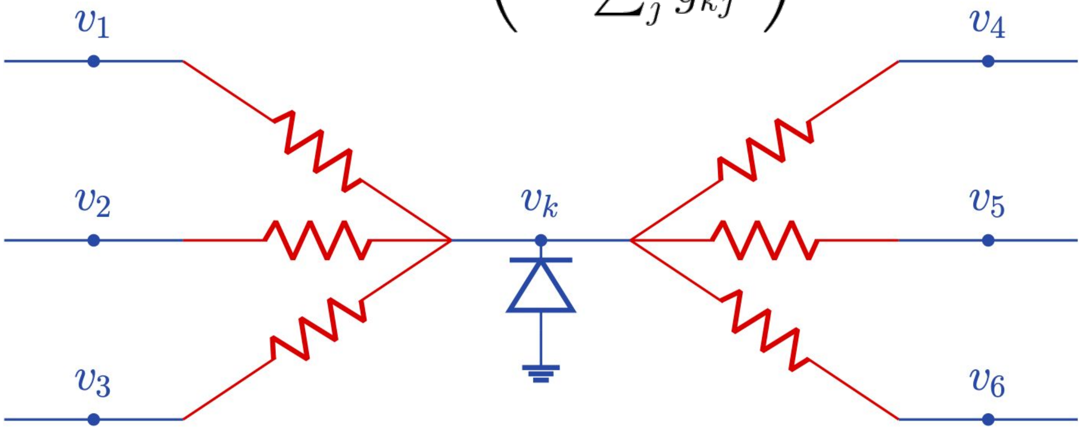
Simulations of deep resistive networks

Main idea:

- assumption: the circuit elements are **ideal**
- the math considerably simplifies

Equation of a node voltage

$$v_k = \max\left(0, \frac{\sum_j g_{kj} v_j}{\sum_j g_{kj}}\right)$$



Reference: “A fast algorithm to simulate nonlinear resistive networks”. To appear.

Simulator for energy-based algorithms

Three key abstractions

Energy function

- deep resistive network (DRN)
- …

Energy minimizer

(in the space of network configurations)

- block coordinate descent
- …

Learning algorithm

(in the weight space)

- equilibrium propagation (positive, negative, centered, …)
- …

Link: Code will be available at https://github.com/rain-neuromorphics/energy-based-learning

Simulator for energy-based algorithms

Three key abstractions

Energy function

- deep resistive network (DRN)
- deep Hopfield network (DHN)
- …

Energy minimizer

(in the space of network configurations)

- block coordinate descent
- gradient descent
- …

Learning algorithm

(in the weight space)

- equilibrium propagation (positive, negative, centered, …)
- contrastive (Hebbian) learning
- coupled learning
- …
- truncated backpropagation (baseline)
- recurrent backpropagation (baseline)

Link: Code will be available at https://github.com/rain-neuromorphics/energy-based-learning

Simulations of Hopfield networks

Scellier and Bengio [1]

MNIST

Ernoult et al. [2]

MNIST
Convolutional HN

Laborieux et al. [3]

CIFAR-10
3 days on a single GPU

Laborieux and Zenke [4]

ImageNet 32*
* downsampled to 32x32 pixels
6 days on 4 GPUs

Scellier et al. [5]

CIFAR-100
3 hours on a single GPU
13.5 faster than [3]

References

[1] Scellier and Bengio. “Equilibrium propagation.” *Frontiers in computational neuroscience* 11 (2017): 24.
[2] Ernoult et al. "Updates of equilibrium prop match gradients of backprop through time." NeurIPS (2019).
[3] Laborieux et al. “Scaling equilibrium propagation to deep ConvNets.” Frontiers in neuroscience (2021).
[4] Laborieux and Zenke. "Holomorphic equilibrium propagation." NeurIPS (2022).
[5] Scellier et al. “Energy-based learning algorithms: a comparative study.” NeurIPS (2023)

Hardware experiments of Hopfield networks

nature electronics

Article https://doi.org/10.1038/s41928-022-00869-w

Activity-difference training of deep neural networks using memristor crossbars

Received: 10 March 2022

Accepted: 13 October 2022

Su-in Yi[1], Jack D. Kendall[2], R. Stanley Williams[1] & Suhas Kumar[3]

- 64 × 64 memristor crossbar array
- Task: classify Braille words
- demonstrate **10,000x improvement** (over GPUs) in energy efficiency for training

Caveat: use additional (SRAM) memory to store the free and nudged states

Reference: Yi et al. "Activity-difference training of deep neural networks using memristor crossbars." Nature Electronics (2023)

Summary

Training deep resistive networks (DRNs) with equilibrium propagation (EP):

1. DRNs are universal function approximators
2. EP performs gradient descent on a cost function
3. DRN simulator is 45,000x faster than SPICE simulations
4. Small-scale experiments demonstrate 10,000x energy efficiency gains over GPUs

Thanks to all my collaborators!

Jack Kendall

Maxence Ernoult

Suhas Kumar

Ross Pantone

RAIN

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Axel Laborieux

Julie Grollier

Damien Querlioz

Yann Ollivier

Siddhartha Mishra

Kalpana Manickavasagam

Vidyesh Anisetti

Ananth Kandala

Jennifer Schwarz

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